LECTURE NO 16

Electrostatics

- Gausses's Law
- Application of Gauss law
- Line charge
- Point charge
- Volume charge
- Basics of Maxwell's equation,

GAUSS'S LAW—MAXWELL'S EQUATION

Gauss's law states that the total electric flux Ψ through any *closed* surface is equal to the total charge enclosed by that surface.

Thus

$$\Psi = Q_{enc} \tag{4.39}$$

that is,

$$\Psi = \oint d\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$= \text{Total charge enclosed } Q = \int \rho_v \, dv \tag{4.40}$$

or

$$\boxed{Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv} \tag{4.41}$$

By applying divergence theorem to the middle term in eqs. (4.41)

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D} \, dv \tag{4.42}$$

Comparing the two volume integrals in eqs. (4.41) and (4.42) results in

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}} \tag{4.43}$$

which is the first of the four *Maxwell's equations* to be derived. Equation (4.43) states that the volume charge density is the same as the divergence of the electric flux density. This should not be surprising to us from the way we defined the divergence of a vector in eq. (3.32) and from the fact that ρ_v at a point is simply the charge per unit volume at that point.

APPLICATIONS OF GAUSS'S LAW

B. Infinite Line Charge

Suppose the infinite line of uniform charge ρ_L C/m lies along the z-axis. To determine **D** at a point P, we choose a cylindrical surface containing P to satisfy symmetry condition as shown in Figure 4.14. **D** is constant on and normal to the cylindrical Gaussian surface; that is, $\mathbf{D} = D_\rho \mathbf{a}_\rho$. If we apply Gauss's law to an arbitrary length ℓ of the line

$$\rho_L \ell = Q = \oint \mathbf{D} \cdot d\mathbf{S} = D_\rho \oint dS = D_\rho 2\pi\rho\ell \qquad (4.46)$$

where $\oint dS = 2\pi\rho\ell$ is the surface area of the Gaussian surface. Note that $\int \mathbf{D} \cdot d\mathbf{S}$ evaluated on the top and bottom surfaces of the cylinder is zero since **D** has no z-component; that means that **D** is tangential to those surfaces. Thus

$$\mathbf{D} = \frac{\rho_L}{2\pi\rho}\,\mathbf{a}_\rho \tag{4.47}$$

as expected from eqs. (4.21) and (4.35).